General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MPC4

Unit Pure Core 4

Monday 24 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 (a) Express $2\sin x + 5\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value of α to the nearest $0.1°$. *(3 marks)*

(b) (i) Write down the maximum value of $2\sin x + 5\cos x$. *(1 mark)*

(ii) Find the value of x in the interval $0° \leqslant x \leqslant 360°$ at which this maximum occurs, giving the value of x to the nearest $0.1°$. *(2 marks)*

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2 (a) The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 9x^3 + 18x^2 - x - 2$.

(i) Use the Factor Theorem to show that $3x + 1$ is a factor of $\mathrm{f}(x)$. *(2 marks)*

(ii) Express $\mathrm{f}(x)$ as a product of three linear factors. *(3 marks)*

(iii) Simplify $\dfrac{9x^3 + 21x^2 + 6x}{\mathrm{f}(x)}$. *(3 marks)*

(b) When the polynomial $9x^3 + px^2 - x - 2$ is divided by $3x - 2$, the remainder is -4.

Find the value of the constant p. *(2 marks)*

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3 (a) Express $\dfrac{3 + 9x}{(1 + x)(3 + 5x)}$ in the form $\dfrac{A}{1 + x} + \dfrac{B}{3 + 5x}$, where A and B are integers. *(3 marks)*

(b) Hence, or otherwise, find the binomial expansion of $\dfrac{3 + 9x}{(1 + x)(3 + 5x)}$ up to and including the term in x^2. *(7 marks)*

(c) Find the range of values of x for which the binomial expansion of $\dfrac{3 + 9x}{(1 + x)(3 + 5x)}$ is valid. *(2 marks)*

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4 A curve is defined by the parametric equations

$$x = 3e^t, \quad y = e^{2t} - e^{-2t}$$

(a) (i) Find the gradient of the curve at the point where $t = 0$. *(3 marks)*

(ii) Find an equation of the tangent to the curve at the point where $t = 0$. *(1 mark)*

(b) Show that the cartesian equation of the curve can be written in the form

$$y = \frac{x^2}{k} - \frac{k}{x^2}$$

where k is an integer. *(2 marks)*

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5 A model for the radioactive decay of a form of iodine is given by

$$m = m_0 2^{-\frac{1}{8}t}$$

The mass of the iodine after t days is m grams. Its initial mass is m_0 grams.

(a) Use the given model to find the mass that remains after 10 grams of this form of iodine have decayed for 14 days, giving your answer to the nearest gram. *(2 marks)*

(b) A mass of m_0 grams of this form of iodine decays to $\frac{m_0}{16}$ grams in d days.

Find the value of d. *(2 marks)*

(c) After n days, a mass of this form of iodine has decayed to less than 1% of its initial mass.

Find the minimum integer value of n. *(3 marks)*

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6 (a) (i) Given that $\tan 2x + \tan x = 0$, show that $\tan x = 0$ or $\tan^2 x = 3$. *(3 marks)*

(ii) Hence find all solutions of $\tan 2x + \tan x = 0$ in the interval $0° < x < 180°$. *(1 mark)*

(b) (i) Given that $\cos x \neq 0$, show that the equation

$$\sin 2x = \cos x \cos 2x$$

can be written in the form

$$2\sin^2 x + 2\sin x - 1 = 0$$ *(3 marks)*

(ii) Show that all solutions of the equation $2\sin^2 x + 2\sin x - 1 = 0$ are given by $\sin x = \dfrac{\sqrt{3} - 1}{p}$, where p is an integer. *(3 marks)*

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7 (a) (i) Solve the differential equation $\dfrac{dx}{dt} = \sqrt{x}\sin\left(\dfrac{t}{2}\right)$ to find x in terms of t. *(3 marks)*

(ii) Given that $x = 1$ when $t = 0$, show that the solution can be written as

$$x = (a - \cos bt)^2$$

where a and b are constants to be found. *(3 marks)*

(b) The height, x metres, above the ground of a car in a fairground ride at time t seconds is modelled by the differential equation $\dfrac{dx}{dt} = \sqrt{x}\sin\left(\dfrac{t}{2}\right)$.

The car is 1 metre above the ground when $t = 0$.

(i) Find the greatest height above the ground reached by the car during the ride. *(2 marks)*

(ii) Find the value of t when the car is first 5 metres above the ground, giving your answer to one decimal place. *(2 marks)*

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8 The coordinates of the points A and B are $(3,\ -2,\ 4)$ and $(6,\ 0,\ 3)$ respectively.

The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

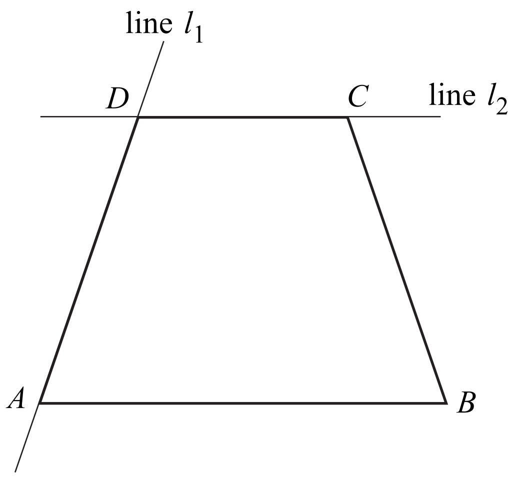
(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Calculate the acute angle between $\overrightarrow{AB}$ and the line l_1, giving your answer to the nearest 0.1°. *(4 marks)*

(b) The point D lies on l_1 where $\lambda = 2$. The line l_2 passes through D and is parallel to AB.

(i) Find a vector equation of line l_2 with parameter μ. *(2 marks)*

(ii) The diagram shows a symmetrical trapezium $ABCD$, with angle DAB equal to angle ABC.



The point C lies on line l_2. The length of AD is equal to the length of BC.

Find the coordinates of C. *(6 marks)*