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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics (Pilot) 6360

XMCA2 Core A2

Mark Scheme

2008 examination - June series

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

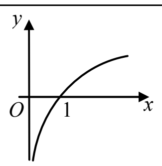
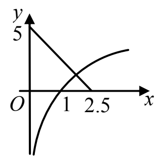
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

XMCA2

Q	Solution	Marks	Total	Comments
1	x 0 0.5 1 1.5 2	B1		PI
	y 1 $\sqrt{2}$ $\sqrt{5}$ $\sqrt{10}$ $\sqrt{17}$	B1		PI
	1 1.41(4) 2.23(6) 3.16(2) 4.12(3)			
	$\frac{1}{3}\times 0.5\{y(0)+y(2)+\ldots$ $\ldots 4[y(0.5)+y(1.5)]+2y(1)\}$	M1		Use of Simpson's rule
	$=\frac{27.90120\ldots}{6}=4.65$ (to 2 d.p.)	A1	4	CAO
	Total		**4**	
2(a)(i)	p(1) = 0	B1	1	
(ii)	$x=\frac{3}{2}$	B1		Seeing $\frac{3}{2}$ OE
	$p(1.5)=6(1.5)^3-7(1.5)^2-11(1.5)+12$	M1		Attempting to evaluate p(1.5) or p(−1.5)
	p(1.5) = 20.25−15.75−16.5+12 = 0 so $(2x-3)$ is a factor of p(x)	A1	3	CSO Need both the arithmetic to show '= 0' and the conclusion.
(b)(i)	$p(x)=(2x-3)(x-1)(3x+4)$	B1		$(x-1)$ a factor or $(3x^2+x-4)$ a factor
		B1	2	Correct product of three linear factors
(ii)	$2x^2-3x=x(2x-3)$	B1		
	$\frac{2x^2-3x}{6x^3-7x^2-11x+12}=\frac{x}{(x-1)(3x+4)}$	B1	2	OE CSO No further cancellation
	Total		**8**	
3(a)	$\{A=\}$ 400	B1	1	
(b)	$640=Ae^{8k}$	M1		
	$8k=\ln\left(\frac{640}{A}\right)$	m1		Use of $e^N=m\Rightarrow N=\ln m$
	$k=\frac{1}{8}\ln 1.6=0.058750(4\ldots)$ $=0.05875$ to 4sf	A1	3	CSO Must see either $\frac{\ln 1.6}{8}$ or at least the 6th dp before seeing the printed answer
(c)	$e^{kt}>\frac{2000}{A}\Rightarrow t>\frac{1}{k}\ln\left(\frac{2000}{A}\right)$	M1		Condone use of '='
	$t>\frac{1}{k}\ln 5$ $t>27.39\ldots$			
	Population will first exceed 2000 in April 2010	A1	2	If T&I, or NMS, mark as B2 or B0
	Total		**6**	

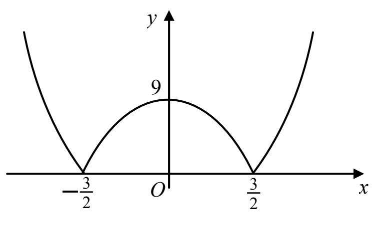
XMCA2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\frac{3+5x}{(1+x)(1+2x)} = \frac{A}{1+x} + \frac{B}{1+2x}$ $\Rightarrow 3 + 5x = A(1+2x) + B(1+x)$	M1		Correct partial fractions form and either multiplication by denominator or cover up rule attempted.
	Substitute $x = -1$; Substitute $x = -0.5$	m1		Either use (any) two values of x to find A and B or equate coefficients to form and attempt to solve $2A+B=5$ and $A+B=3$
	$A = 2$, $B = 1$	A1	3	NMS mark as B3 or B0
(b)(i)	$(1+x)^{-1} = 1 + (-1)x + px^2 + qx^3$	M1		$p \neq 0$
	$= 1 - x + x^2 - x^3 ...$	A1	2	
(ii)	$(1+2x)^{-1} = 1 + -(2x) + (2x)^2 - (2x)^3$	M1		Replacing x by $2x$ or start again to get expansion (at least 3 terms)
	$\frac{3+5x}{(1+x)(1+2x)} = 2(1+x)^{-1} + (1+2x)^{-1}$	M1		Using (a) with powers '−1'. PI
	$= 2(1 - x + x^2 - x^3 ...) + …$ $…(1-2x + 4x^2 - 8x^3 …)$	m1		Dep. on previous 3Ms
	$= 3 - 4x + 6x^2 - 10x^3$	A1	4	Award equivalent marks for other valid methods
	Total		**9**	
5(a)		B1		General shape with no part of the graph in 2nd and 3rd quadrants
		B1	2	Crossing x-axis at 1 and asymptotic to y-axis **SC** Graph correct in 1st quadrant but stopping at $x = 1$ (labelled), award B1
(b)(i)		B1		Identifying and drawing line $y = 5-2x$ on c's sketch of $y = \ln x$.
	Since the line $y = 5-2x$ only intersects the curve $y = \ln x$ at one point (the equation $\ln x = 5 - 2x$ has only one root).	E1	2	
(ii)	Let $f(x) = \ln x - 5 + 2x$ $f(2.1) = -0.05(8…)$ $f(2.2) = 0.1(8…)$	M1		Both f(2.1) and f(2.2) attempted **or** subst. of x=2.1 and x=2.2 in given equation
	Since change of sign (and f is continuous), root α lies between 2.1 and 2.2	A1	2	OE comparing sides of given eqn for the 2 vals. of x with concl.
(c)	$x_2 = 2.1290$	B1		AWRT 2.1290 ; condone 2.129
	$x_3 = 2.1222$	B1		AWRT 2.1222
	$x_4 = 2.1238$	B1	3	CAO
(d)	$\ln x = 5 - 2x \Rightarrow x = e^{5-2x}$ $\Rightarrow x = e^5 \times e^{-2x} \Rightarrow xe^{2x} = e^5$	M1		Accept either direction
	Since α is the root of $\ln x = 5 - 2x$, α must also be the root of $xe^{2x} = e^5$.	A1	2	Working must be full and correct and conclusion explicit. Verification by calculator on its own scores 0/2
	Total		**11**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$R = 5$	B1		OE
	$\cos\alpha = \frac{4}{R}$, $\sin\alpha = \frac{3}{R}$ or $\tan\alpha = \frac{3}{4}$	M1		PI
	α = 0.644 and R = 5	A1	3	Accept α = 0.643(50…)
(b)	5	B1√	1	Accept 'R'
(c)	Translation $\begin{bmatrix} -0.644... \\ 0 \end{bmatrix}$	B1 B1√		Translation-transl..with no variable comp $\begin{bmatrix} -0.643... \\ 0 \end{bmatrix}$ OE Ft on c's α. Accept $\begin{bmatrix} -\alpha \\ 0 \end{bmatrix}$
	Stretch, **(I)** parallel to y-axis, **(II)** scale factor 5.	M1 A1√	4	'Stretch' with either **(I)** or **(II)** Complete description. Ft on c's R (Accept 'R' for '5')
	Total		**8**	
7(a)	$\sec^4 x - \tan^4 x = \sec^4 x - (\sec^2 x - 1)^2$ $= \sec^4 x - (\sec^4 x - 2\sec^2 x + 1)$	M1 M1		Use of $1+\tan^2 x = \sec^2 x$ OE identity [Award this M1 if cand factorises and reaches $\sec^4 x - \tan^4 x = \sec^2 x + \tan^2 x$]
	$= 2\sec^2 x - 1$	A1	3	AG CSO
(b)	…. $\Rightarrow 2\sec^2 x - 1 = 31 \Rightarrow \sec^2 x = 16$	M1		
	$x = \cos^{-1}(0.25)$, $x = \cos^{-1}(-0.25)$	m1		OE PI Accept either for m1
	= 75.5°, 104.5°, 255.5°, 284.5°.	A2,1	4	A1 for any two correct. −1 for each additional (>4) answer in the given range. Ignore answers outside range
	Total		**7**	
8(a)	$u = x$ and $\frac{dv}{dx} = e^{2x}$	M1		Attempt to use parts formula in the 'correct direction'
	$\frac{du}{dx} = 1$ and $v = \frac{1}{2}e^{2x}$	A1		PI
	$\ldots = x\,\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx$	A1√		ft on wrong integration of e^{2x} provided v is of the form ke^{2x}
	$= \frac{1}{2}x\,e^{2x} - \frac{1}{4}e^{2x}$ $(+c)$	A1	4	CSO (Condone absence of $+c$)
(b)	$u = 1 + \ln 2x$			
	$du = \frac{2}{2x}\,dx$	M1		"$du = f(x)\,dx$" or "$dx = g(x)\,du$"
	$\int \frac{1}{x(1+\ln 2x)}\,dx = \int \frac{1}{u}\,du$	m1 A1		Eliminating all x's
	$\{\ln u\ (+c)\}$			
	$\ldots = \ln(1+\ln 2x)$ $(+c)$	A1√	4	ft on $k\int \frac{1}{u}\,du$
	Total		**8**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
9	$\frac{dy}{dx} = \frac{(4x^2+3)(e^x) - e^x(8x)}{(4x^2+3)^2}$	M1 A1		Quotient rule OE
	For a stationary point $\frac{dy}{dx} = 0 \Rightarrow e^x(4x^2 - 8x + 3) = 0$	m1		Must have scored earlier M1 and reached the stage of e^x times a three term quadratic equals 0.
	$e^x \neq 0$	E1		OE statement
	The equation has exactly 2 (real) roots (x =0.5;1.5); ($b^2 - 4ac = 16$) so curve has exactly two stationary points.	M1 E1	 6	Use of $b^2 - 4ac > 0$ or method for solving $ax^2+bx+c=0$, with non-zero a, b and c. Completion with conclusion dep on prev M1
	Total		**6**	
10(a)	$f(x) \geq -9$	B1	1	or $y \geq -9$ or $f \geq -9$ or ≥ -9 but <u>not</u> $x \geq -9$
(b)(i)	$fg(x) = \frac{4}{(x-1)^2} - 9$	B1	1	ACF
(ii)	$fg(x) = 16 \Rightarrow \frac{4}{(x-1)^2} = 25$	M1		Puts (b)(i) = 16 and performs a relevant rearrangement.
	$(x-1)^2 = \frac{4}{25}$			
	$x - 1 = \pm\frac{2}{5}$	m1		Reaching stage where just one step required to find x. (± not required)
	$\Rightarrow x = \frac{3}{5}, \frac{7}{5}$	A1	3	Need both NMS Mark as B3 or B0
(c)	$y = g^{-1}(x) \Rightarrow g(y) = x$			
	$\Rightarrow \frac{1}{y-1} = x$	M1		$x \leftrightarrow y$ at any stage
	$\Rightarrow y - 1 = \frac{1}{x}$	M1		Into a form where just one step is required
	$g^{-1}(x) = 1 + \frac{1}{x}$	A1	3	ACF [Accept y in place of $g^{-1}(x)$]
(d)(i)		M1 A1 A1	 3	Modulus graph (−1.5, 0) (1.5, 0) (0, 9) OE Values on axes accepted Correct shape including cusps
(ii)	$0 < k < 9$	B2,1√	2	Ft c's pt of intersection with +'ve y-axis. B1 for $0 \leq k \leq$ '9' or $0 < k \leq$ '9' or $0 \leq k <$ '9' or $k <$ '9'
	Total		**13**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
11	$\int y \, \mathrm{d}y = \int (x+1)^5 \, \mathrm{d}x$	M1		Separating variables with intention to then integrate.
	$\frac{y^2}{2} = \frac{(x+1)^6}{6} \quad (+c)$	A1A1		
	$\frac{1}{2} = \frac{1}{6} + c$	m1		Substituting $x = 0$, $y = 1$ to find c
	$c = \frac{1}{3}$ so $3y^2 = (x+1)^6 + 2$	A1	5	ACF. Condone finishing at $c = \frac{1}{3}$ provided no earlier errors.
	Total		**5**	
12(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 8t \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = 4 - 6t^2$	M1		Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ attempted with at least one correct.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1		Chain rule
	$= \frac{4 - 6t^2}{8t}$	A1	3	ACF but must be in terms of t. NMS B3 or B0
(b)	When $t = 1$ $x = 4(1)^2 + 1, \quad y = 4(1) - 2(1)^3$	M1		Sub t=1 to find either (x **and** y) or ($\mathrm{d}y/\mathrm{d}x$)
	$x = 5 \quad y = 2$	A1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{8}$	A1√		Ft on (a)
	Eqn of tangent: $y - 2 = -\frac{2}{8}(x - 5)$	A1√		Any correct ft form. OE eg $\frac{0-2}{x_P - 5}$ = cand's $\frac{-2}{8}$
	At P, $y = 0 \Rightarrow x = 13$	A1	5	
(c)	$y^2 = 16t^2 - 16t^4 + 4t^6$			
	$y^2 = 4(x-1) - (x-1)^2 + 4\left(\frac{x-1}{4}\right)^3$	M1		Squaring y (must be correct number of terms) and elimination of t
		A1	2	ACF of f(x) in y^2 = f(x)
	Total		**10**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
13(a)	Putting **r**'s equal $1-\lambda=-1+\mu$ $2+\lambda=-2+3\mu$ $3-2\lambda=14-4\mu$	M1		Or any two components
	Solving two equations 1st&2nd $\lambda=0.5,\ \mu=1.5$; [2nd &3rd $\lambda=-8.5,\ \mu=-1.5$] [1st &3rd $\lambda=-0.5,\ \mu=2.5$]	m1 A1		 Any correct pair
	Check in remaining equation	m1		Attempt to use c's solutions to check remaining equation or attempt to solve another relevant pair of equations and to compare solution(s). Not dependent on previous m
	Since equation is not satisfied, lines do not intersect	A1	5	CSO Fully correct solution with valid conclusion
(b)	$\begin{bmatrix}-1\\1\\-2\end{bmatrix}\neq k\begin{bmatrix}1\\3\\-4\end{bmatrix}$ so lines not parallel. (*)			
	Lines do not intersect and are not parallel so l_1 and l_2 are skew lines. (**)	E2,1	2	E1 if either just (*) or just (**)
(c)	$\overrightarrow{AP}=\begin{bmatrix}1-p\\2+p\\3-2p\end{bmatrix}-\begin{bmatrix}3\\-1\\2\end{bmatrix}$	M1		Condone if reversed ie $\pm\begin{bmatrix}-2-p\\3+p\\1-2p\end{bmatrix}$ seen.
	$\overrightarrow{AP}\bullet\begin{bmatrix}-1\\1\\-2\end{bmatrix}=0$	M1		
	$2+p+3+p-2+4p=0$	A1		Condone one slip in simplifying $\overrightarrow{AP}$
	$p=-0.5$	A1	4	CSO
	Total		**11**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
14(a)	$\cos(x + 2x) = \cos x\cos 2x - \sin x\sin 2x$	M1		
	$= \cos x\,(2\cos^2 x - 1) - \sin x(2\sin x\cos x)$	B1B1		Double angle identities ACF seen
	$= \cos x(2\cos^2 x - 1) - 2\cos x(1 - \cos^2 x)$	A1		Correct unsimplified, all in cos x
	$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$			
	$\cos 3x = 4\cos^3 x - 3\cos x$	A1	5	CSO AG
(b)	$V = \pi\int_0^{\frac{\pi}{2}} 4\cos^3 x\ \mathrm{d}x$	M1		$V = (\pi)\int\ y^2\ \mathrm{d}x$ with y in terms of x
	$= \pi\int_0^{\frac{\pi}{2}} (\cos 3x + 3\cos x)\ \mathrm{d}x$	m1		Using part (a)
	$= \pi\left[\frac{1}{3}\sin 3x + 3\sin x\right]_0^{\frac{\pi}{2}}$	m1		Reverse 'Chain rule', $(\pm\frac{1}{3}\sin 3x)$
		A1		Terms inside [ ]
	$= \pi\left(-\frac{1}{3} + 3\right) = \frac{8}{3}\pi$	A1	5	CSO Award equivalent marks for other valid methods
	Total		**10**	
15	$2x \qquad\qquad = 0$	B1		Must be an equation and not have a spurious '$\frac{\mathrm{d}y}{\mathrm{d}x} =$' at the start. (Can be retrieved if clear recovery later)
	$+\left(y + x\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	M1 A1		Use of product rule
	$-2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$-3\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \Rightarrow y = 3x - 3$	M1		Setting $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ to obtain an equation in y and x only (can be unsimplified)
	$x^2 + x(3x-3) - (3x-3)^2 - 3(3x-3) = 1$	m1		Substituting into eqn. of curve to obtain an eqn. in x (or in y) only
	$5x^2 - 6x + 1 = 0 \qquad [5y^2 + 12y = 0]$ $(5x - 1)(x - 1) = 0 \Rightarrow x = 0.2,\ x = 1$	m1		Valid method to find x (or y). Dep on previous two marks)
	$(0.2, -2.4)$ and $(1, 0)$	A1	9	CSO
	Total		**9**	
	TOTAL		**125**	