

General Certificate of Education
June 2009
Advanced Level Examination



MATHEMATICS (PILOT)
Unit Core A2

XMCA2

Friday 19 June 2009 1.30 pm to 4.00 pm

For this paper you must have:

- a 16-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 2 hours 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is XMCA2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 125.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The polynomial $p(x)$ is defined by $p(x) = 2x^4 + 3x^3 - 8x^2 - 14x - 3$.

(a) Use the Factor Theorem to show that $2x + 3$ is a factor of $p(x)$. (3 marks)

(b) It is given that $p(x) = (2x + 3)(x^3 - 4x - 1)$.

(i) Show that, for $x > 0$, the equation $x^3 - 4x - 1 = 0$ can be rearranged into the form $x = \sqrt{\frac{1}{x} + 4}$. (2 marks)

(ii) Use the iteration $x_{n+1} = \sqrt{\frac{1}{x_n} + 4}$ with $x_1 = 2$ to find the values of x_2 , x_3 and x_4 , giving your answers to four decimal places. (3 marks)

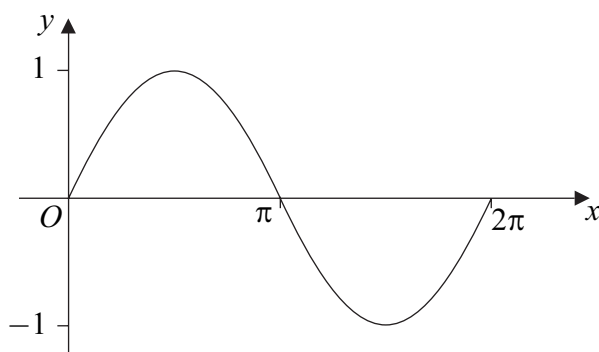
2 (a) Express $\frac{5+x}{(1-x)(2+x)}$ in the form $\frac{A}{1-x} + \frac{B}{2+x}$, where A and B are integers. (3 marks)

(b) Find the binomial expansion of:

(i) $(1-x)^{-1}$ up to and including the term in x^2 ; (2 marks)

(ii) $\frac{5+x}{(1-x)(2+x)}$ up to and including the term in x^2 . (5 marks)

- 3 The diagram shows the graph of $y = \sin x$ for $0 \leq x \leq 2\pi$.



- (a) (i) Sketch the graph of $y = |\sin x|$ in the interval $0 \leq x \leq 2\pi$. (2 marks)
- (ii) The equation $|\sin x| = k$, where k is a constant, has exactly two solutions in the interval $0 \leq x \leq 2\pi$. State the value of k . (1 mark)
- (b) Sketch the graph of $y = \operatorname{cosec} x$ in the interval $0 < x < 2\pi$. (2 marks)

- 4 A curve has equation $y = \frac{e^{3x}}{x+2}$.

- (a) Find $\frac{dy}{dx}$. (3 marks)
- (b) The curve intersects the y -axis at the point A . Find an equation of the tangent to the curve at the point A . (4 marks)

- 5 A curve C has equation $y = \sqrt{\cos(x^2)}$, where x is in radians. When the region bounded by the curve C , the coordinate axes and the line $x = 1$ is rotated through 2π radians about the x -axis, the volume generated is V .

Use Simpson's rule with five ordinates (four strips) to find an approximation to V , giving your answer to four decimal places. (6 marks)

6 The function f is defined with its domain by

$$f(x) = \ln(2x + 3), \quad x \geq 0$$

- (a) (i) Sketch the graph of $y = f(x)$. *(2 marks)*
- (ii) Write down the range of f . *(2 marks)*
- (b) The inverse of f is f^{-1} .
- (i) Find an expression for $f^{-1}(x)$. *(3 marks)*
- (ii) State the domain of f^{-1} . *(1 mark)*
- (c) Differentiate $\ln(2x + 3)$ with respect to x . *(2 marks)*
- (d) The curves $y = f(x)$ and $y = f^{-1}(x)$ intersect at the point P . The x -coordinate of P is α .
- (i) Show that $e^\alpha = 2\alpha + 3$. *(3 marks)*
- (ii) Hence verify that the product of the gradients of the curves at P is 1. *(2 marks)*

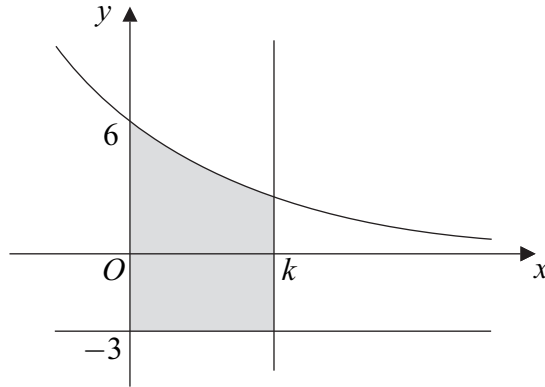
- 7 (a) Prove that the curve with equation $y = xe^x$ has exactly one stationary point and find its coordinates. *(6 marks)*
- (b) The x -axis is a tangent to the curve with equation $y = xe^x + k$, where k is a constant. Determine the exact value of k . *(2 marks)*

8 Solve the differential equation

$$\frac{dy}{dx} = \frac{y \cos x}{6 + \sin x}$$

given that $y = 2$ when $x = 0$. Give your answer in the form $y = f(x)$, where $f(x)$ is in its simplest form. *(5 marks)*

- 9 (a) Describe a sequence of two geometrical transformations which maps the graph of $y = e^{2x}$ onto the graph of $y = 6e^{-2x}$. (4 marks)
- (b) The diagram shows the curve with equation $y = 6e^{-2x}$ and the lines $y = -3$ and $x = k$, where k is a positive constant.



The shaded region is bounded by the curve $y = 6e^{-2x}$, the y -axis and the lines $y = -3$ and $x = k$.

The area of the shaded region is 4 square units.

- (i) Show that k satisfies the equation $(3k - 1)e^{2k} - 3 = 0$. (6 marks)
- (ii) Show that $0.6 < k < 0.7$. (2 marks)

- 10 The points A and B have coordinates $(2, 0, 0)$ and $(5, 1, 4)$ respectively.

The line l passes through the point A and has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

The acute angle between the line AB and the line l is θ .

- (a) Find a vector equation for the line AB , using a parameter λ . (3 marks)
- (b) Show that $\cos \theta = \frac{9}{2\sqrt{39}}$. (4 marks)
- (c) The perpendicular from B to the line l meets l at the point P .
- (i) Find the coordinates of P . (5 marks)
- (ii) Given that P is the mid-point of BC , find the coordinates of C . (2 marks)

Turn over ►

- 11 (a) By using the expansion of $\sin(A + B)$, prove the identity

$$\sin 3x = 3 \sin x - 4 \sin^3 x \quad (5 \text{ marks})$$

- (b) Hence solve the equation

$$2 \sin 3x = 1 - \cos 2x$$

in the interval $0^\circ < x < 360^\circ$, giving all solutions to the nearest 0.1° where appropriate.
(7 marks)

- 12 (a) (i) Find $\int x \sec^2 x \, dx$. (4 marks)

(ii) Hence find $\int x \tan^2 x \, dx$. (2 marks)

- (b) Use the substitution $x = 2 \sin \theta$ to find $\int \sqrt{4 - x^2} \, dx$, giving your answer in terms of x . (6 marks)

- 13 The curve C is defined by the parametric equations

$$x = 3t + t^3, \quad y = 8 - 3t^2$$

Prove that the normal to C at the point $P(-4, 5)$ does not cut the curve C again.
(11 marks)

END OF QUESTIONS

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