

General Certificate of Education
June 2009
Advanced Level Examination



MATHEMATICS
Unit Pure Core 4

MPC4

Thursday 11 June 2009 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Use the Remainder Theorem to find the remainder when $3x^3 + 8x^2 - 3x - 5$ is divided by $3x - 1$. (2 marks)
- (b) Express $\frac{3x^3 + 8x^2 - 3x - 5}{3x - 1}$ in the form $ax^2 + bx + \frac{c}{3x - 1}$, where a , b and c are integers. (3 marks)

- 2 A curve is defined by the parametric equations

$$x = \frac{1}{t}, \quad y = t + \frac{1}{2t}$$

- (a) Find $\frac{dy}{dx}$ in terms of t . (4 marks)
- (b) Find an equation of the normal to the curve at the point where $t = 1$. (4 marks)
- (c) Show that the cartesian equation of the curve can be written in the form

$$x^2 - 2xy + k = 0$$

where k is an integer. (3 marks)

- 3 (a) Find the binomial expansion of $(1 - x)^{-1}$ up to and including the term in x^2 . (2 marks)
- (b) (i) Express $\frac{3x - 1}{(1 - x)(2 - 3x)}$ in the form $\frac{A}{1 - x} + \frac{B}{2 - 3x}$, where A and B are integers. (3 marks)
- (ii) Find the binomial expansion of $\frac{3x - 1}{(1 - x)(2 - 3x)}$ up to and including the term in x^2 . (6 marks)
- (c) Find the range of values of x for which the binomial expansion of $\frac{3x - 1}{(1 - x)(2 - 3x)}$ is valid. (2 marks)

- 4 A car depreciates in value according to the model

$$V = Ak^t$$

where $\pounds V$ is the value of the car t months from when it was new, and A and k are constants. Its value when new was $\pounds 12\,499$ and 36 months later its value was $\pounds 7000$.

- (a) (i) Write down the value of A . (1 mark)
- (ii) Show that the value of k is 0.984 025, correct to six decimal places. (2 marks)
- (b) The value of this car first dropped below $\pounds 5000$ during the n th month from new. Find the value of n . (3 marks)

- 5 A curve is defined by the equation $4x^2 + y^2 = 4 + 3xy$.

Find the gradient at the point $(1, 3)$ on this curve. (5 marks)

- 6 (a) (i) Show that the equation $3 \cos 2x + 7 \cos x + 5 = 0$ can be written in the form $a \cos^2 x + b \cos x + c = 0$, where a , b and c are integers. (3 marks)
- (ii) Hence find the possible values of $\cos x$. (2 marks)
- (b) (i) Express $7 \sin \theta + 3 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. Give your value of α to the nearest 0.1° . (3 marks)
- (ii) Hence solve the equation $7 \sin \theta + 3 \cos \theta = 4$ for all solutions in the interval $0^\circ \leq \theta \leq 360^\circ$, giving θ to the nearest 0.1° . (3 marks)
- (c) (i) Given that β is an acute angle and that $\tan \beta = 2\sqrt{2}$, show that $\cos \beta = \frac{1}{3}$. (2 marks)
- (ii) Hence show that $\sin 2\beta = p\sqrt{2}$, where p is a rational number. (2 marks)

Turn over for the next question

Turn over ►

7 The points A and B have coordinates $(3, -2, 5)$ and $(4, 0, 1)$ respectively.

The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$.

(a) Find the distance between the points A and B . (2 marks)

(b) Verify that B lies on l_1 . (2 marks)

(c) The line l_2 passes through A and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 3 \\ -8 \end{bmatrix}$.

The lines l_1 and l_2 intersect at the point C . Show that the points A , B and C form an isosceles triangle. (6 marks)

8 (a) Solve the differential equation

$$\frac{dx}{dt} = \frac{150 \cos 2t}{x}$$

given that $x = 20$ when $t = \frac{\pi}{4}$, giving your solution in the form $x^2 = f(t)$. (6 marks)

(b) The oscillations of a ‘baby bouncy cradle’ are modelled by the differential equation

$$\frac{dx}{dt} = \frac{150 \cos 2t}{x}$$

where x cm is the height of the cradle above its base t seconds after the cradle begins to oscillate.

Given that the cradle is 20 cm above its base at time $t = \frac{\pi}{4}$ seconds, find:

(i) the height of the cradle above its base 13 seconds after it starts oscillating, giving your answer to the nearest millimetre; (2 marks)

(ii) the time at which the cradle will first be 11 cm above its base, giving your answer to the nearest tenth of a second. (2 marks)

END OF QUESTIONS