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General Certificate of Education

Mathematics (Pilot) 6360

XMCA2 Core A2

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

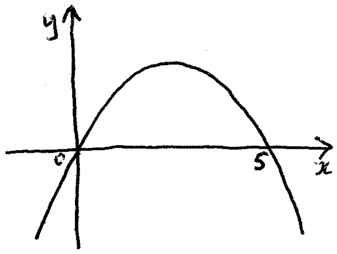
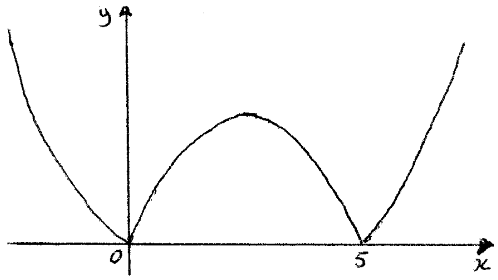
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

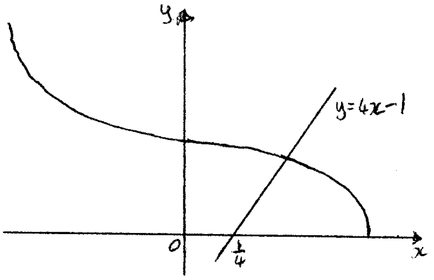
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

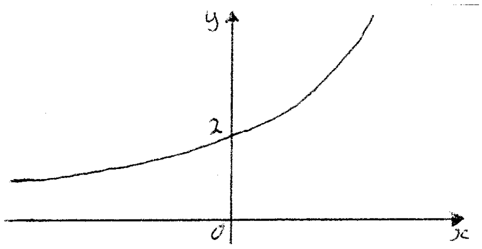
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

XMCA2

Q	Solution	Marks	Total	Comments
1(a)(i)		B1		Inverted parabola passing through the origin and positive x-axis
(ii)		M1		Modulus graph
		A1	3	Correct shape including cusps and meeting the x-axis at origin (0,0) and (5,0)…accept 5 on x-axis as being equivalent
(b)	$5x - x^2 = 6$ so $x = 2,\ x = 3$	B1		Both values of x
	$5x - x^2 = -6$ $x^2 - 5x - 6 = 0$	M1		OE
	$(x - 6)(x + 1) = 0$	m1		OE method to solve quadratic
	$x = 6,\ x = -1$	A1	4	
	Total		**7**	
2(a)	$\frac{dx}{dt} = 3$, $\frac{dy}{dt} = 12t$	M1		Both derivatives attempted with at least one correct. PI
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 4t$	M1		Chain rule
	$= 4$ at P	A1		
	When $t = 1$, $x = 5$, $y = 6$ $P(5, 6)$	B1		Both
	Tangent: $y - 6 = 4(x - 5)$	A1F		OE
	$y = 4x - 14$	A1	6	
(b)	$(x-2)^2 = 9t^2 = 9\left(\frac{y}{6}\right)$	M1		Elimination of t
	$y = \frac{2}{3}(x-2)^2$	A1	2	CSO ACF of f(x).
	Total		**8**	

Q	Solution	Marks	Total	Comments
3(a)	$A(-1, \pi)$; $B(1, 0)$	B1 B1	2	
(b)(i)		B1		Line (positive gradient, positive x-intercept closer to O than B) intersecting the curve.
	Since the line $y = 4x - 1$ only intersects the curve $y = \cos^{-1} x$ at one point (the equation $\cos^{-1}x = 4x - 1$ has only one root).	E1	2	Eqn of line must be explicit somewhere in (b)(i)
(ii)	Let $f(x) = \cos^{-1}x - 4x + 1$ $f(0.5) = 0.04(7\ldots)$ $f(0.6) = -0.4(7\ldots)$	M1		Both f(0.5) and f(0.6) attempted **or** subst of $x = 0.5$ and $x = 0.6$ in given equation
	Since change of sign (and f is continuous), root α lies between 0.5 and 0.6	A1	2	OE comparing sides of given eqn. for the two values of x with a valid conclusion with explicit reference to 0.5 and 0.6 A0 if incorrect values to at least 1sf
(c)	$x_2 = 0.512$ $x_3 = 0.508$ $x_4 = 0.509$	B1 B1 B1	3	AWRT 0.512 AWRT 0.508 CAO
(d)	x — y 0 — 1.570796… 0.2 — 1.369438… 0.4 — 1.159279… 0.6 — 0.927295… 0.8 — 0.643501…	B1 B1		x values PI y values PI; award correct to 2dp values if seen. Accept $\frac{\pi}{2}$ for $y(0)$
	$\frac{1}{3}\times 0.2\{y(0) + y(0.8) + 4[y(0.2) + y(0.6)] + 2y(0.4)\}$	M1		Use of Simpson's rule
	$\frac{1}{3}\times 0.2\times 13.71979\ldots = 0.9147$ (to 4 d.p.)	A1	4	CAO <u>Note</u> For calcs set in degree mode, maximum possible is B1B0M1A0
	Total		**13**	

Q	Solution	Marks	Total	Comments
4(a)		B1		Shape and graph not below x-axis
		B1	2	Intersects y-axis at 2 Accept just as a statement
(b)(i)	$\frac{dy}{dx} = \frac{(1+e^{2x})(-\sin x) - \cos x(2e^{2x})}{(1+e^{2x})^2}$	B1 M1		Differentiating e^{2x} as $2e^{2x}$ Quotient rule used OE Condone sign errors
		A1F		Ft on c's derivative of e^{2x}
	When $x = 0$, $\frac{dy}{dx} = -\frac{1}{2}$	A1	4	
(ii)	When $x = 0$, $\frac{dx}{dy} = -2$	B1F	1	Ft on 1/[c's answer to (b)(i)]
	Total		**7**	
5(a)	$6x^2 + 10 = A(x+1)^2 + B(3x-1)$	M1		OE Ignore extra term $C\,(x+1)(3x-1)$
	$x = \frac{1}{3}$ $x = -1$	m1		Subst values of x or comparing coefficients PI
	$A = 6$ $B = -4$	A2, (1F)	4	C must be found if included earlier. Ft value of one constant if used to find other(s)
(b)	$\int \left(\frac{6}{3x-1} - \frac{4}{(x+1)^2} \right) dx$	M1		Use of (**a**)
	$= \frac{6}{3}\ln(3x-1)$	m1 A1F		$k\ln(3x-1)$ Condone missing brackets Ft on c's A
	$-4\left(-\frac{1}{x+1}\right)$	B1F		Ft on c's B
	$\int_1^3 \frac{6x^2+10}{(3x-1)(x+1)^2}\,dx = (2\ln 8 + 1) - (2\ln 2 + 2)$	m1		F(3) − F(1)
	$= 4\ln 2 - 1$	A1	6	CAO Accept $p = 4$, $q = -1$
	Total		**10**	

Q	Solution	Marks	Total	Comments
6(a)	$f(x) \ge 2$ (accept $y \ge 2$)	M1 A1	 2	>2 ; ≥2 ; f ≥2 [but x≥2 is M0] SC f(x)≥2, f(x)≤−2 (B1)
(b)	$\text{gf}(x) = \dfrac{1}{\sqrt{(x+4)}+2}$	B1	1	ACF ISW after ACF
(c)	$y = f^{-1}(x) \Rightarrow f(y) = x \Rightarrow \sqrt{y+4} = x$ $y + 4 = x^2$ $f^{-1}(x) = x^2 - 4$	M1 M1 A1	 3	$x \leftrightarrow y$ at any stage Squaring to eliminate the square root ACF Accept if left as y in place of $f^{-1}(x)$.
	Total		**6**	
7(a)(i)	$p\left(\dfrac{1}{3}\right) = 0$	B1	1	
(ii)	$p(-0.5) =$ $30(-0.5)^3 - (-0.5)^2 - 6(-0.5) + 1$ $p(-0.5) = -3.75 - 0.25 + 3 + 1 = 0$ so $(2x+1)$ is a factor of $p(x)$.	 M1 A1	 2	 Attempting to evaluate p(−0.5) CSO Need both the arithmetic to show '=0' and the conclusion
(iii)	$p(x) = (2x + 1)(15x^2 - 8x + 1)$ $= (2x + 1)(3x - 1)(5x - 1)$	B1 M1 A1	 3	$(3x - 1)$ a factor or $(15x^2 - 8x + 1)$ a factor Valid method to find the third linear factor. PI Correct product of three linear factors
(b)	$\cot 2\theta = \dfrac{1-t^2}{2t}$ $\sec^2\theta = 1 + \tan^2\theta = 1 + t^2$ $\dfrac{1-t^2}{2t} = 18 - 15(1+t^2)$ $1 - t^2 = 6t - 30t^3$ $30t^3 - t^2 - 6t + 1 = 0$	B1 B1 M1 A1	 4	[In (b), condone if left as $\tan\theta$ for t] Stated or used Stated or used All in terms of t (or $\tan\theta$) CSO AG
(c)	$\Rightarrow (2\tan\theta + 1)(3\tan\theta - 1)(5\tan\theta - 1) = 0$ $\tan\theta = -0.5$, $\tan\theta = 1/3$; $\tan\theta = 1/5$ $\theta = 11.3°, 18.4°, 153.4°$	M1 A2,1,0	 3	Use of parts (**a**) and (**b**) A1 if either one correct or 3 correct to the nearest degree. Condone greater accuracy
	Total		**13**	

Q	Solution	Marks	Total	Comments
8(a)	$\int 2x\ln x\,\mathrm{d}x \quad u = \ln x \quad \frac{\mathrm{d}v}{\mathrm{d}x} = 2x$	M1		Attempt to use integration by parts in the 'correct direction'
	$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}, \quad v = x^2$	A1		PI
	$\int 2x\ln x\,\mathrm{d}x = (\ln x)\,x^2 - \int x^2 \times \frac{1}{x}\,\mathrm{d}x$	A1F		Ft on one error in above line
	$= x^2\ln x - \frac{x^2}{2}\ (+c)$	A1	4	CSO Condone absence of $+c$
(b)	Let $u = 2x+1$ '$\mathrm{d}u = 2\,\mathrm{d}x$'	M1 m1		A relevant single substitution used
	$\int 4x^2\sqrt{2x+1}\,\mathrm{d}x = \int (u-1)^2\sqrt{u}\,\frac{1}{2}\mathrm{d}u$	m1		In terms of u only. Dep on prev 2 mks
	$= \frac{1}{2}\int (u^{2.5} - 2u^{1.5} + u^{0.5})\,\mathrm{d}u$	m1		Integrand into a form which can be integrated directly. Dep on prev 3 mks
	$= \frac{1}{2}\left(\frac{u^{3.5}}{3.5} - \frac{2u^{2.5}}{2.5} + \frac{u^{1.5}}{1.5}\right)\ (+c)$			
	$= \frac{(2x+1)^{3.5}}{7} - \frac{2(2x+1)^{2.5}}{5} + \frac{(2x+1)^{1.5}}{3}\ (+c)$	A1	5	CSO Condone absence of $+c$ Altn substitution: $u^2 = 2x+1$ so '$2u\mathrm{d}u = 2\,\mathrm{d}x$' (M1m1) leading to $\int u^2(u^2-1)^2\,\mathrm{d}u$ (m1) leading to $\int (u^6 - 2u^4 + u^2)\,\mathrm{d}u$ (m1) If uses $t = x$ and then integration by parts twice, full marks are still possible.
	Total		**9**	

Q	Solution	Marks	Total	Comments
9(a)	$\frac{dV}{dt} = kV^{\frac{1}{3}}$	M1		$\frac{dV}{dt}$ seen
		A1		ACF
	$\int V^{-\frac{1}{3}}\,dV = \int k\,dt$	m1		Separating variables with intention to then integrate
	$\frac{3}{2}V^{\frac{2}{3}} = kt \quad +c$	A1		Condone absence of $+c$ for this mark
	$\frac{3}{2}\times 8000^{\frac{2}{3}} = k\times 0 + c$ $c = \frac{3}{2}\times 400$	m1		Substituting $V = 8000$ and $t = 0$ in an attempt to find c.
	$\frac{3}{2}V^{\frac{2}{3}} = kt + 600 \Rightarrow 3V^{\frac{2}{3}} = 2kt + 1200$	A1	6	CSO AG
(b)	$3\times 27000^{\frac{2}{3}} = 2k\times 8 + 1200$ $k = 93.75$	M1		Substituting $V = 27000$ and $t = 8$ in an attempt to find k.
	$V^{\frac{2}{3}} = \frac{2\times 93.75\times 13 + 1200}{3}$ =1212.5	A1		
	$V = 1212.5^{1.5} =$ £42 200 (to the nearest £100)	A1	3	Condone either £42220 or £42220.42 or £42220.43
	Total		**9**	

Q	Solution	Marks	Total	Comments
10(a)	$(1+kx)^{-2} = 1-2kx$	M1		
	$+3k^2x^2 - 4k^3x^3 ....$	A1	2	
(b)(i)	$(1+kx)^{-2} - (1+x)^n =$			
	$-2kx - nx$			
	$+3k^2x^2 - \dfrac{n(n-1)}{2}x^2$			
	$-4k^3x^3 - \dfrac{n(n-1)(n-2)}{3!}x^3$	M1		Applies the two expansions at least as far as terms in x.
	$-2k - n = 0 \Rightarrow n = -2k$	A1	2	AG
(ii)				Accept work for (ii) which appears in candidate's working for (i)
	$3k^2 - \dfrac{n(n-1)}{2} = 6$	M1		Applies the two expansions and attempts to equate the combined coefficient of x^2 to 6.
		A1		
	$n = -2k$ so $3k^2 + k(-2k-1) = 6$	m1		Eliminates n (or k) to obtain a quadratic in k (or n) only
	$k^2 - k - 6 = 0$; $(k-3)(k+2) = 0$	A1		OE if used 'formula'. If quadr in n then $(n+6)(n-4) = 0$
	Since $n<0$, $k = 3$ and $n = -6$	A1		Both needed
	Coeff of x^3, $p = -4(3)^3 - \dfrac{(-6)(-7)(-8)}{6} = -52$	A1	6	CSO
	Total		**10**	

Q	Solution	Marks	Total	Comments
11(a)	Putting **r**'s equal $1 + 2\lambda = 24 + 12\mu$ $2 + \lambda = 9 + 3\mu$ $3 - 2\lambda = 4 + 4\mu$	M1 A1		Or any two components Three correct equations
	Solving two equations	m1		
	$\lambda = 2.5,\ \mu = -1.5$;	A1		
	Check in remaining equation	m1		Attempt to use c's solutions to check remaining equation or attempt to solve another relevant pair of equations and to compare solution(s). Not dependent on previous m
	B (6, 4.5, −2)	A1	6	CSO Condone equivalent column vector
(b)	Let angle between the given lines be θ. Let $\boldsymbol{d}_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ and $\boldsymbol{d}_2 = \begin{bmatrix} 12 \\ 3 \\ 4 \end{bmatrix}$			Could be using multiples of these direction vectors eg could be using $\boldsymbol{d}_1 = \overrightarrow{AB}$
	$\cos\theta = \dfrac{d_1 \bullet d_2}{\lvert d_1\rvert\ \lvert d_2\rvert}$	M1		Must be **using** this formula with relevant vectors
	$\lvert \boldsymbol{d}_1 \rvert = \sqrt{2^2 + 1^2 + (-2)^2}$ (= 3) $\lvert \boldsymbol{d}_2 \rvert = \sqrt{12^2 + 3^2 + 4^2}$ (= 13)	m1		Either. OE
	$\boldsymbol{d}_1 \bullet \boldsymbol{d}_2 = 24 + 3 - 8 = 19$ $\cos\theta = \dfrac{19}{39}$ (=0.487..); θ = 60.8°	m1		OE
	Since angle between the lines is not 60° it is not possible to find a point P on l_2 such that triangle ABP is equilateral.	A1	4	CSO including a clear correct explanation/conclusion. [Accept $\cos\theta = \dfrac{19}{39} \neq \dfrac{1}{2}$ as alternative to seeing 60.8°]
	Total		**10**	

<u>Comparing lengths of sides AB, BP, AP to show triangle is not equilateral:</u>

$AB^2=5^2+2.5^2+5^2$ $\{=7.5^2\}$; $BP^2=(18+12\mu)^2+(4.5+3\mu)^2+(6+14\mu)^2$; $AP^2=(23+12\mu)^2+(7+3\mu)^2+(1+4\mu)^2$ **M1 for one; +m1 for three**

$BP=AB$ gives $169\mu^2+507\mu+324=0$; $(13\mu+27)(13\mu+12) = 0$;

When $\mu=-27/13$, $AP^2=57.69..\neq 56.25$; When $\mu=-12/13$, $AP^2=167.3..\neq 56.25$ so cannot find a pt P on l_2 such that triangle ABP is equilateral.

m1 Equating two of the three lengths, forming equation and solving to find value(s) of μ and comparing lengths of sides

A1CSO fully correct solution with clear correct explanation/conclusion

Q	Solution	Marks	Total	Comments
12(a)	$R = 10$; $\sin\alpha = 0.6$	B1; B1F	2	ft on $\sin\alpha = 6/R$ for $R>8$
(b)	Translation	B1		Translation (the translation must not contain a variable component)
	$\begin{bmatrix} 0.643... \\ 0 \end{bmatrix}$	B1		In place of 0.643.. accept 'α' or AWRT '0.64' or c's ft value for α from (a). Vector can be replaced by its equivalent in words
	Stretch with scale factor 10	M1		Accept 'R' or c's value of R in (a) in place of '10' but must have both 'Stretch' and the scale factor
	parallel to y-axis	A1	4	
(c)	[Can award the 1st 5 marks in (c) even if limits are missing or incorrect]			
	$V = \pi\int_0^\alpha (8\cos x + 6\sin x)^2 \, dx$	B1		$\pi\int (8\cos x + 6\sin x)^2 \, dx$ PI
	$= (\pi)\int_0^\alpha [10\cos(x-\alpha)]^2 \, dx$	B1F		ft on c's R in (a). π not required now until last mark.
	$= (\pi)\int_0^\alpha \left\{100 \times \frac{1}{2}\left[1+\cos(2x-2\alpha)\right]\right\} dx$	M1 A1		$\cos^2(x-\alpha)$ in terms of $\cos(2x-2\alpha)$
	$= (\pi) \times 50 \times \left[x + \frac{1}{2}\sin(2x-2\alpha)\right]_0^\alpha$	A1F		Correct integration of $K[1+\cos(2x-2\alpha)]$. Ft on wrong K
	$= (\pi) \times 50 \times \left[(\alpha+0) - [0 + \frac{1}{2}\sin(-2\alpha)]\right]$	m1		$F(\alpha) - F(0)$
	$= (\pi) \times 50 \times \left[\alpha + \frac{1}{2}\sin 2\alpha\right]$			
	$= (\pi) \times 50 \times [\alpha + \sin\alpha\cos\alpha]$	m1		Valid method to find an **exact** value for $\sin(-2\alpha)$ PI by correct ans
	$= 50\pi[\alpha + 0.6 \times 0.8] = 50\pi\alpha + 24\pi = 2\pi(25\alpha + 12)$	A1	8	CSO
	Altn to (c)…..			
	$V = \pi\int_0^\alpha (8\cos x + 6\sin x)^2 \, dx$	(B1)		$\pi\int (8\cos x + 6\sin x)^2 \, dx$ PI
	$(\pi)\int_0^\alpha [64\cos^2 x + 96\sin x\cos x + 36\sin^2 x] \, dx$	(B1)		ACF π not required
	$(\pi)\int_0^\alpha [28 \times \frac{1}{2}(1+\cos 2x) + 48\sin 2x + 36] \, dx$	(M1; M1)		M1 for $\cos^2 x$ <u>and</u> $\sin^2 x$ terms in terms of $\cos 2x$; M1 for valid start to integrate $\sin x\cos x$
	$= (\pi)[50x + 7\sin 2x - 24\cos 2x]_0^\alpha$	(A1F)		Correct integration only ft wrong non-zero coeffs
	$(\pi)[(50\alpha + 7\sin 2\alpha - 24\cos 2\alpha) - (-24)]$	(m1)		$F(\alpha) - F(0)$
	$(\pi)[50\alpha + 14 \times 0.6 \times 0.8 - 24(2 \times 0.8^2 - 1) + 24]$	(m1)		Finding exact vals of $\sin 2\alpha$ and $\cos 2\alpha$
	$= \pi[50\alpha + 6.72 - 6.72 + 24] = 2\pi(25\alpha + 12)$	(A1)		CSO
	Total		**14**	

Q	Solution	Marks	Total	Comments
13	$+6x^2 = 0$	B1		Must be an equation and not have a spurious '$\frac{dy}{dx} =$ 'at the start. (Can be retrieved if clear recovery later)
	$9y^2\frac{dy}{dx}$	B1		
	$-6\left(y + x\frac{dy}{dx}\right)$	M1 A1		Use of product rule
	For St. Pts. $\frac{dy}{dx} = 0 \Rightarrow y = x^2$	M1		Setting $\frac{dy}{dx} = 0$ to obtain an equation in y **and** x only (condone k present)
	$3(x^2)^3 - 6x(x^2) + 2x^3 = k$	m1		Substituting into eqn. of curve to obtain an eqn. in x (or in y) and k only
	$3x^6 - 4x^3 - k = 0$ $[3y^3 - 4y^{1.5} - k = 0]$	A1		ACF but simplified
	$b^2 - 4ac = 16 + 12k$	m1		Recognises equation is quadratic and considers the discriminant either explicitly or as part of the quadratic formula solutions
	If $k < -\frac{4}{3}$, $b^2 - 4ac < 0$ so roots are not real. In the case when $k < -\frac{4}{3}$, there are no real values of x for which $\frac{dy}{dx} = 0$ so the curve has no stationary points if $k < -\frac{4}{3}$.	A1	9	CSO AG Needs to be fully correct with a clear justified conclusion explicitly stated.
	Total		**9**	
	TOTAL		**125**	