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General Certificate of Education

Mathematics (Pilot) 6360

XMCA2 Core A2

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

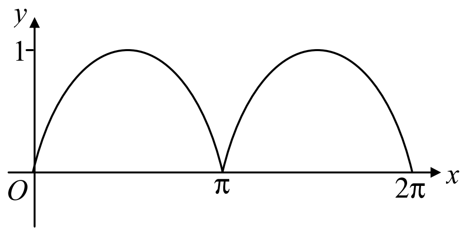
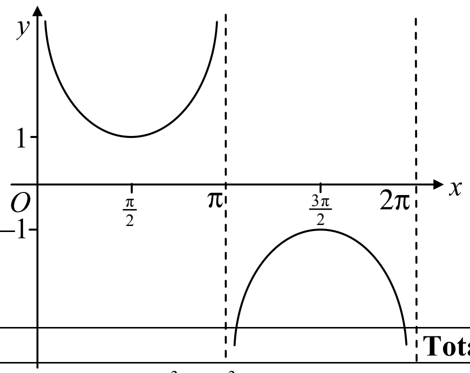
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

XMCA2

Q	Solution	Marks	Total	Comments
1(a)	$x = -\frac{3}{2}$	B1		Seeing $-\frac{3}{2}$ OE
	$p(-1.5) = 2(-1.5)^4 + 3(-1.5)^3 - 8(-1.5)^2 - 14(-1.5) - 3$	M1		Attempting to evaluate p(−1.5)
	p(−1.5) = 10.125−10.125−18+21−3 = 0 so $(2x + 3)$ is a factor of $p(x)$	A1	3	CSO Need both the arithmetic to show '= 0' and the conclusion.
(b)(i)	$x^3 - 4x - 1 = 0 \Rightarrow x(x^2 - 4) - 1 = 0 \Rightarrow x^2 - 4 = \frac{1}{x}$	M1		Dividing throughout by x OE
	$x^2 = \frac{1}{x} + 4 \Rightarrow x = \sqrt{\frac{1}{x} + 4}$ (since $x > 0$)	A1	2	AG CSO
(ii)	$x_2 = 2.1213$	B1		AWRT 2.121
	$x_3 = 2.1146$	B1		AWRT 2.1146
	$x_4 = 2.1149$	B1	3	CAO
	Total		**8**	
2(a)	$5 + x = A(2 + x) + B(1 - x)$	M1		
	Substitute $x = 1$; Substitute $x = -2$	m1		Either use (any) two values of x to find A and B or equate coeffs to form and attempt to solve $A-B=1$ and $2A+B=5$ condone wrong coefficients.
	$A = 2$, $B = 1$	A1	3	NMS or cover up rule: 3/3 if both values correct; 1/3 if only one value correct
(b)(i)	$(1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2$	M1		$1 \pm x + px^2$, $p \neq 0$
	$= 1 + x + x^2 ...$	A1	2	Fully simplified
(ii)	$\frac{2}{1-x} = 2(1 + x + x^2 + ...)$	B1F		Ft on A and expn of $(1-x)^{-1}$
	$\frac{1}{2+x} = 2^{-1}\left[1 + \frac{x}{2}\right]^{-1}$	B1		
	$\left[1 + \frac{x}{2}\right]^{-1} = \left[1 + (-1)\left(\frac{x}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2 + ...\right]$	M1		$[1 + (-1)\left(\frac{x}{2}\right) + kx^2]$, $k \neq 0$
	$= 1 - \frac{x}{2} + \frac{x^2}{4} + \ldots$			
	$\frac{5+x}{(1-x)(2+x)} = \frac{A}{1-x} + \frac{B}{2+x} = 2(1-x)^{-1} + (2+x)^{-1}$	M1		Valid combination of both expansions
	$= 2(1 + x + x^2 ...) + \frac{1}{2}\left(1 - \frac{x}{2} + \frac{x^2}{4} + ...\right) =$			
	$\frac{5}{2} + \frac{7}{4}x + \frac{17}{8}x^2$	A1	5	Award equivalent marks for other valid methods
	Total		**10**	

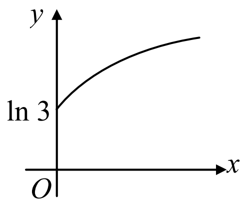
XMCA2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)		M1 A1	 2	Modulus graph Correct shape including cusp at (π, 0). Ignore any part of graph beyond $0 \le x \le 2\pi$.
(ii)	$k = 1$	B1	1	
(b)		M1 A1	 2	Two branch curve, general shape correct. Min at $(\alpha, 1)$ Max at $(\beta, -1)$ with α roughly halfway between 0 and π, and β roughly halfway between π and 2π and curve asymptotic to $x = 0$, $x = \pi$ and $x = 2\pi$.
	Total		**5**	
4(a)	$\frac{dy}{dx} = \frac{(x+2)3e^{3x} - e^{3x}(1)}{(x+2)^2}$	B1 M1 A1	 3	$\frac{d}{dx}\left(e^{3x}\right) = 3e^{3x}$ Quotient rule OE
(b)	When $x = 0$, $\frac{dy}{dx} = \frac{6e^0 - e^0}{2^2} = \frac{5}{4}$ $A\left(0, \frac{1}{2}\right)$ Equation of tangent at A: $y - \frac{1}{2} = \frac{5}{4}(x - 0)$	M1 A1F B1 A1	 4	Attempt to find $\frac{dy}{dx}$ at $x=0$ Dep. only on the previous M $x = 0$, $y = 0.5$ ACF
	Total		**7**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
5	$V = \pi\int_0^1 \cos(x^2)\ \mathrm{d}x$	M1		$\int \cos(x^2)\ (\mathrm{d}x)$. Condone missing brackets around x^2.
		A1		Correct limits PI. (Condone $k\pi$ or missing π until the final mark)
	Applying Simpson's rule to $\int_0^1 \cos(x^2)\ \mathrm{d}x$			
	x 0 0.25 0.5 0.75 1	B1		PI
	$Y{=}y^2$ 1 0.9980(47) 0.9689(12) 0.8459(24) 0.5403(02)	B1		PI Accept 3dp or better
	[πY vals. 3.1415(9) 3.1354(5) 3.0439(2) 2.6575(5) 1.6974(0)]			
	$\frac{0.25}{3}\times\{Y(0)+Y(1)+ 4[Y(0.25)+Y(0.75)]+2Y(0.5)\}$	M1		Use of Simpson's rule
	$V = \pi \times \frac{10.85401....}{12}$ So $V = 2.8416$ (to 4 d.p.)	A1	6	CAO
	Total		**6**	

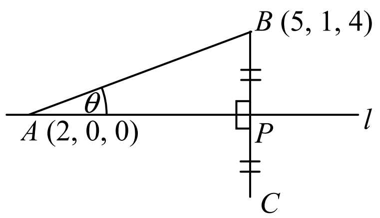
XMCA2 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)		B2,1,0	2	B2 correct sketch-no part of curve in 2nd,3rd or 4th quadrants and 'ln3' (B1 for correct shape in 1st quadrant, ignore other quadrants; ln3 not required)
(ii)	Range of f: $f(x) \geq \ln 3$	M1 A1	 2	$\geq \ln 3$ or $> \ln 3$ For M mark allow 1.1 for ln3 but dec loses A mark Allow either y or f for $f(x)$
(b)(i)	$y = f^{-1}(x) \Rightarrow f(y) = x$ $\Rightarrow \ln(2y+3) = x$ $\Rightarrow 2y + 3 = e^x$ $f^{-1}(x) = \dfrac{e^x - 3}{2}$	 M1 M1 A1	 3	 $x \Leftrightarrow y$ at any stage Use of $\ln m = N \Rightarrow m = e^N$ ACF; accept y in place of $f^{-1}(x)$
(ii)	Domain of f^{-1} is: $x \geq \ln 3$	B1F	1	ft on (a)(ii) for RHS but must have x with ft inequality sign; or the correct answer
(c)	$\dfrac{d}{dx}[(\ln(2x+3)] = \dfrac{1}{2x+3} \times 2$	M1 A1	 2	$\dfrac{1}{2x+3}$
(d)(i)	P, the pt of intersection of $y = f(x)$ and $y = f^{-1}(x)$, must lie on the line $y = x$; so P has coordinates (α, α). $f(\alpha) = \alpha$ $\ln(2\alpha + 3) = \alpha \Rightarrow 2\alpha + 3 = e^{\alpha}$	M1 M1 A1	 3	 OE eg $f^{-1}(\alpha) = \alpha$ AG CSO $\dfrac{e^{\alpha} - 3}{2} = \alpha \Rightarrow e^{\alpha} = 2\alpha + 3$ if using the OE above.
(ii)	$\dfrac{d}{dx}\left[f^{-1}(x)\right] = \dfrac{1}{2} e^x$ Product of gradients = $\dfrac{e^x}{2x+3}$ At $P(\alpha, \alpha)$, the product of the gradients is $\dfrac{e^{\alpha}}{2\alpha+3} = \dfrac{2\alpha+3}{2\alpha+3} = 1$	B1F B1	 2	 AG CSO
	Total		**15**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{dy}{dx} = x e^x + e^x$	M1 A1		M1 Product rule OE.
	At stationary point(s) $e^x(x+1) = 0$	m1		Equating their $\frac{dy}{dx}$ to 0 PI
	$e^x > 0$	E1		OE eg accept $e^x \neq 0$
	Only one value of x for st. pt. Curve has exactly one st. pt.	A1		CSO with conclusion
	Stationary point is $(-1, -e^{-1})$	A1	6	Condone AWRT −0.368 for $-e^{-1}$
(b)	Stationary point is $(-1, k - e^{-1})$	B1F		Or E1 for $y = x e^x$ to $y = x e^x + k$ is a vertical translation of k units.
	St. pt is on x-axis, so $k = e^{-1}$.	B1	2	NMS 2/2
	Total		**8**	
8	$\int \frac{1}{y}\, dy = \int \frac{\cos x}{6+\sin x}\, dx$	B1		Correct separation; must see dy and dx but condone missing integral signs
	$\ln y = \ln(6 + \sin x)$ $(+c)$	B1 B1		B1 for each side. Condone missing '+c'
	$\ln 2 = \ln 6 + c$	M1		Substituting $x = 0$, $y = 2$ to find c following an integration
	$\ln y = \ln(6 + \sin x) + \ln 2 - \ln 6$			
	so $y = \frac{1}{3}(6+\sin x)$	A1	5	Correct simplified form not involving logs
	Total		**5**	
9(a)	$y = e^{2x} \rightarrow e^{-2x} \rightarrow 6e^{-2x}$			
	Reflection; in the y-axis	M1;A1		
	Stretch,	M1		M1 'Stretch' with either **(I)** or **(II).**
	(I) parallel to y-axis, **(II)** scale factor 6.	A1	4	
				For correct alternatives to the stretch after writing $y = e^{-2x+\ln 6}$, award B1 for 'translation in x-dirn.' and B1 for the correct vector (OE) noting order of transformations.
(b)(i)	Area of rectangle/shaded region below x-axis $= 3k$	B1		
	Area of shaded region above x-axis			
	$= \int_0^k 6e^{-2x}\, dx = \left[-3e^{-2x}\right]_0^k$	B1		$-3e^{-2x}$
	$= -3e^{-2k} - (-3)$	M1		F(k) − F(0) following an integration
		A1		ACF
	Total area of shaded region $= 3k - 3e^{-2k} + 3 = 4$	M1		No remaining integration; ft on above areas
	$3k-1-3e^{-2k} = 0 \Rightarrow (3k-1)e^{2k} - 3 = 0$	A1	6	AG CSO
(ii)	Let $f(k) = (3k-1)e^{2k} - 3$			
	$f(0.6) = 0.8e^{1.2} - 3 = -0.3(4...) < 0$			
	$f(0.7) = 1.1e^{1.4} - 3 = 1.(46...) > 0$	M1		Both f(0.6) and f(0.7) [or better] attempted
	Since change of sign (and f continuous), $0.6 < k < 0.7$	A1	2	AG Note: Must see the explicit reference to 0.6 and 0.7 otherwise A0
	Total		**12**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
10(a)	$(\overrightarrow{AB} =)\quad \begin{bmatrix}5\\1\\4\end{bmatrix}-\begin{bmatrix}2\\0\\0\end{bmatrix}=\begin{bmatrix}3\\1\\4\end{bmatrix}$	M1 A1		M1 for $\pm(\overrightarrow{OB}-\overrightarrow{OA})$ OE for $\overrightarrow{BA}$
	$(\boldsymbol{r}=)\quad \begin{bmatrix}2\\0\\0\end{bmatrix}+\lambda\begin{bmatrix}3\\1\\4\end{bmatrix}$	B1F	3	OE Ft on $\overrightarrow{AB}$
(b)	$\begin{bmatrix}3\\1\\4\end{bmatrix}\bullet\begin{bmatrix}1\\2\\1\end{bmatrix}=3+2+4=9$	M1		$\pm\overrightarrow{AB}\ \bullet$ direction vector of l evaluated
	$\sqrt{3^2+1^2+4^2}=\sqrt{26}$; $\sqrt{1^2+2^2+1^2}=\sqrt{6}$	B1F		Either; Ft on either of c's vectors
	$\sqrt{26}\sqrt{6}\cos\theta=9$	M1		Consistent use of $\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta=\mathbf{a}\bullet\mathbf{b}$
	$\cos\theta=\dfrac{9}{\sqrt{26}\sqrt{6}}=\dfrac{9}{2\sqrt{39}}$	A1	4	AG CSO
(c)(i)				
	$\overrightarrow{BP}=\begin{bmatrix}2+p\\2p\\p\end{bmatrix}-\begin{bmatrix}5\\1\\4\end{bmatrix}=\begin{bmatrix}p-3\\2p-1\\p-4\end{bmatrix}$	M1 A1		Condone one slip
	$\overrightarrow{BP}\bullet\begin{bmatrix}1\\2\\1\end{bmatrix}=0;\quad 6p=9\Rightarrow p=1.5$	M1 A1		"$\pm\overrightarrow{BP}\ \bullet$ direction vector of $l=0$". Condone one slip
	$P\ (3.5, 3, 1.5)$	A1	5	
(ii)	(3.5, 3, 1.5) is mid point of BC so $\dfrac{x_C+5}{2}=3.5\quad\dfrac{y_C+1}{2}=3\quad\dfrac{z_C+4}{2}=1.5$	M1		
	$\Rightarrow\ C\ (2, 5, -1)$	A1F	2	Ft on $(2x_P-5, 2y_P-1, 2z_P-4)$ Condone written as a column vector. Award equivalent marks for alternative valid methods.
	Total		**14**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
11(a)	$\sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$	M1		
	$= [2\sin x\cos x]\cos x + [1-2\sin^2 x]\sin x$	B1;B1		B1 for each […]. Accept alternative correct forms for $\cos 2x$
	$= 2\sin x(1-\sin^2 x) + (1-2\sin^2 x)\sin x$	m1		All in terms of $\sin x$
	$= 2\sin x - 2\sin^3 x - \sin x - 2\sin^3 x$			
	$\sin 3x = 3\sin x - 4\sin^3 x$	A1	5	AG CSO
(b)	$2\sin 3x = 1 - \cos 2x$			
	$2(3\sin x - 4\sin^3 x) = 1 - \cos 2x$	M1		Using (a)
	$2(3\sin x - 4\sin^3 x) = 1 - (1-2\sin^2 x)$	M1		Equation in $\sin x$
		A1		
	$2\sin x\,(3 - \sin x - 4\sin^2 x) = 0$			
	$[2\sin x = 0]\quad (3 - 4\sin x)(1 + \sin x) = 0$	m1		Factorising/solving quadratic in $\sin x$
	$\sin x = 0$; $x = 180°$	B1		Extras in given interval lose rel B/A mrk(s)
	$\sin x = 0.75$; $x = 48.6°, 131.4°$	A1		AWRT
	$\sin x = -1$; $x = 270°$	A1	7	Ignore solns outside $0°<x<360°$ throughout
	Total		**12**	
12(a)(i)	$u = x$ and $\dfrac{dv}{dx} = \sec^2 x$	M1		Attempt to use parts formula in the 'correct direction'
	$\dfrac{du}{dx} = 1$ and $v = \tan x$	A1		PI
	$\ldots\ldots = x\tan x - \int \tan x\,dx$	A1		
	$= x\tan x - \ln(\sec x)\quad (+c)$	A1	4	OE CSO (Condone absence of $+c$)
(ii)	$\int x\tan^2 x\,dx = \int x(\sec^2 x - 1)\,dx$	M1		Use of identity $1 + \tan^2 x = \sec^2 x$
	$= [x\tan x - \ln(\sec x)] - \frac{1}{2}x^2\quad (+c)$	A1F	2	[…] ft on (a)(i)
(b)	$x = 2\sin\theta,\quad dx = 2\cos\theta\,d\theta$	M1		"$dx = f(\theta)\,d\theta$ " OE
	$\int \sqrt{4-x^2}\,dx = \int \sqrt{4(1-\sin^2\theta)}\;2\cos\theta\,d\theta$	m1 A1		Eliminating all x's
	$= \int 4\cos^2\theta\,d\theta = \int 2(\cos 2\theta + 1)\,d\theta$	m1		Use of $\cos 2\theta$ to integrate $\cos^2\theta$
	$= \sin 2\theta + 2\theta\quad (+c)$	A1F		Ft a slip
	$= 2\sin\theta\sqrt{1-\sin^2\theta} + 2\theta\quad (+c)$			
	$= x\sqrt{\left(1-\dfrac{x^2}{4}\right)} + 2\sin^{-1}\left(\dfrac{x}{2}\right)\quad (+c)$	A1	6	ACF (accept unsimplified)
	Total		**12**	

XMCA2 (cont)

Q	Solution	Marks	Total	Comments
13	$x=3t+t^3$ $\quad y=8-3t^2$			
	$\frac{dx}{dt}=3+3t^2$ $\quad \frac{dy}{dt}=-6t$	M1		Both attempted and at least one correct.
	$\frac{dy}{dx}=\frac{-6t}{3+3t^2}$	M1 A1		Chain rule
	At $P(-4, 5)$, $t=-1$	B1		
	At $P(-4, 5)$, $\frac{dy}{dx}=\frac{6}{3+3}=1$ Gradient of normal at P is -1	M1		
	Eqn of normal at P: $y-5=-1(x+4)$ $y+x=1$	A1		ACF
	Normal cuts curve C when $8-3t^2+3t+t^3=1$ $\Rightarrow t^3-3t^2+3t+7=0$	M1 A1		
	$\Rightarrow (t+1)(t^2-4t+7)=0$ (*)	m1		Factorise the cubic
	$(t^2-4t+7)=0$ has no real solutions since $(-4)^2<4(1)(7)$.	M1		
	$t=-1$ is only real solution of (*) so normal only cuts C at P, where $t=-1$ ie the normal does not cut C again.	E1	11	Statement must be correct and fully justified
	Total		**11**	
	TOTAL		**125**	